Utilizing Distance Distribution in Determining Topological Characteristics of Multi-hop Wireless Networks

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Content

- Motivation
- Contribution
- Link Probability
- Topological Characteristics
- Conclusion
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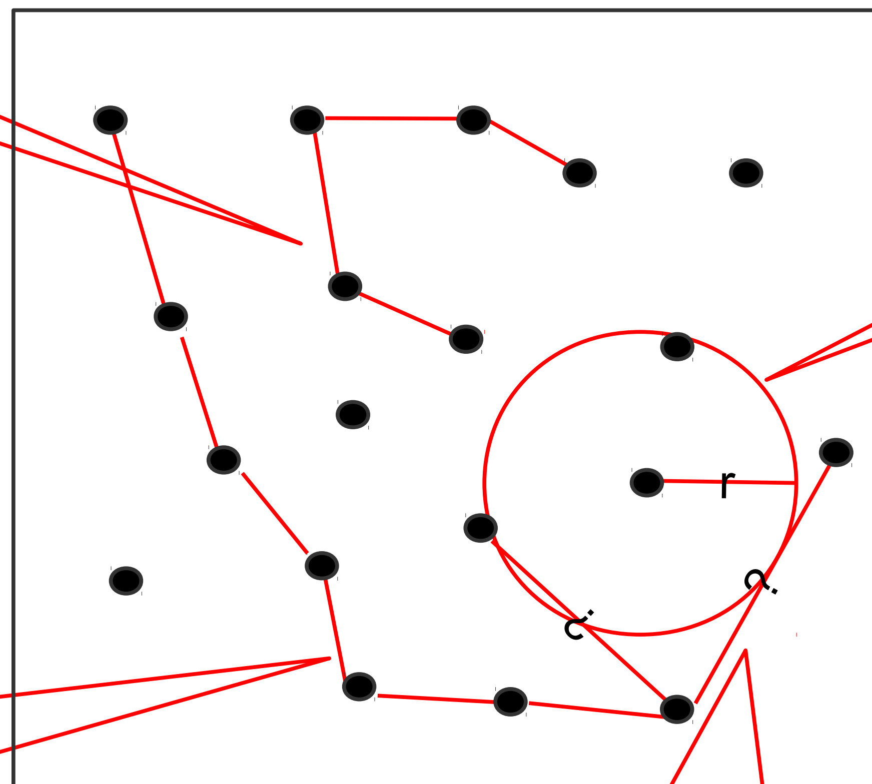
Topological Characteristics

Average shortest path

Degree:# of nodes in communication range



Link Probability
Degree
Average shortest path
Diameter,
Similarly for 3D

Diameter: Longest shortest path

Are there any link?
What is probability?

Importance of Topology Characteristics

- **Performance of Protocol**
  - Diameter
    - Bounds the maximum delay in message communication
  - Average Shortest Path
    - How efficient data transmission
- **Security**
  - Degree
    - Higher degree means higher node connectivity
- **Generate more realistic topologies for Simulations**

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Contribution

- **Developed Analytical Formulas for 2D and 3D**
  - Link Probability
  - Diameter
  - Average Shortest Path
  - Degree

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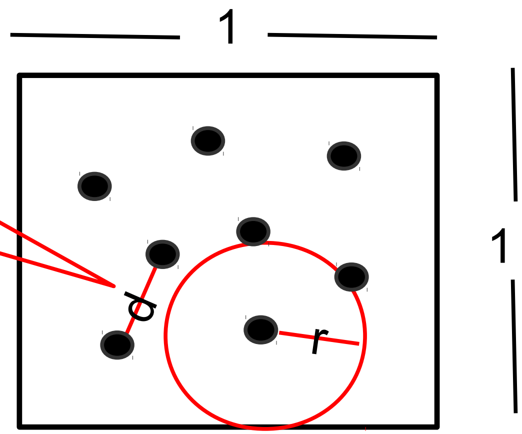
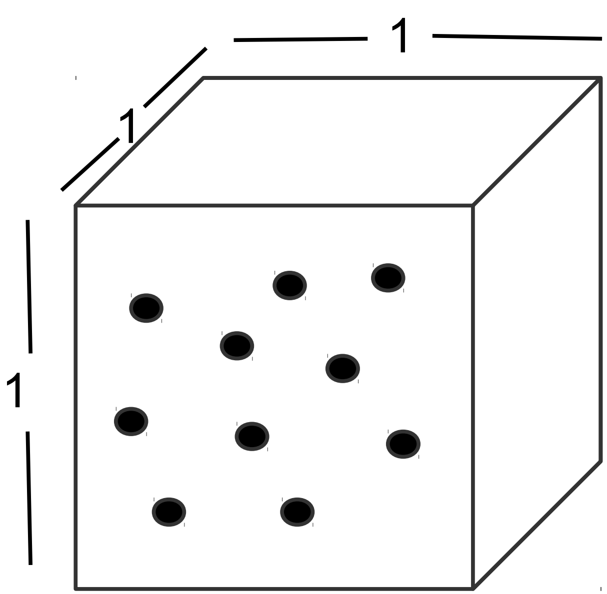
Link Probability



Probability of
d < r

$$F_{2D} = r^2\left(\frac{r^2}{2} - \frac{8r}{3} + \pi\right) \quad where \quad 0 \leqslant r \leqslant 1$$

Probability of d < r
by using distance distribution



$$f_{3D} = 4t^2 - 6\pi t^3 + 8t^4 - t^5$$

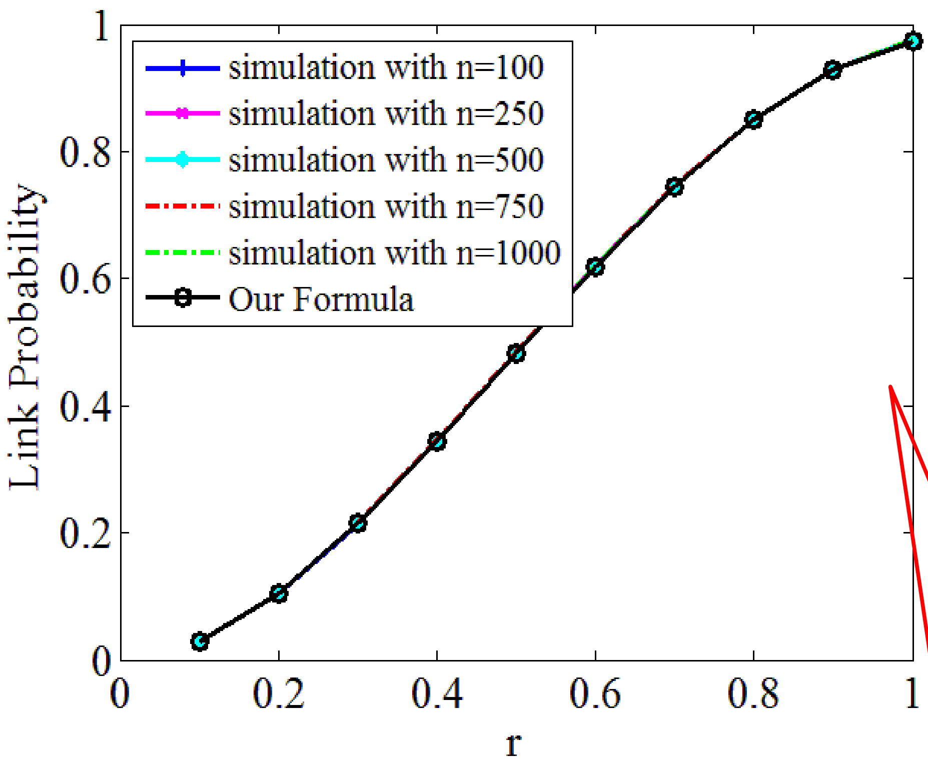
Distance distribution
of unit cube between
0 and 1

$$F_{3D} = \int_0^r f_{3D}(t)\,dt$$

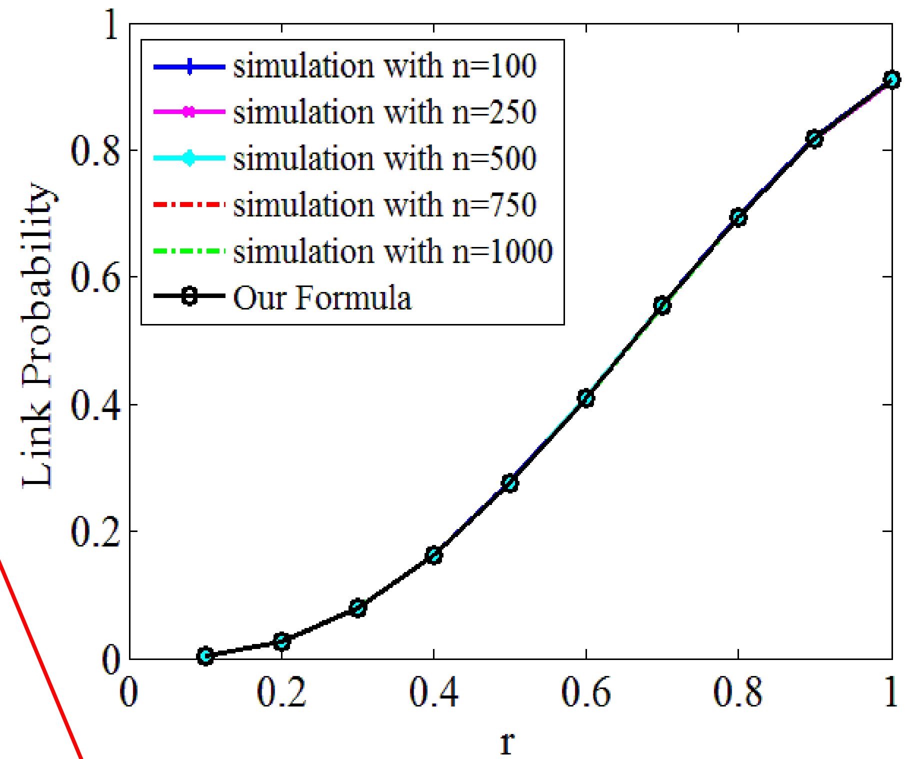
$$F_{3D} = \frac{4\pi r^3}{3} - \frac{6\pi r^4}{4} + \frac{8r^5}{5} - \frac{r^6}{6} \quad where \quad 0 \leqslant r \leqslant 1$$

Link probability
or probability of d < r

Test of Link Probability



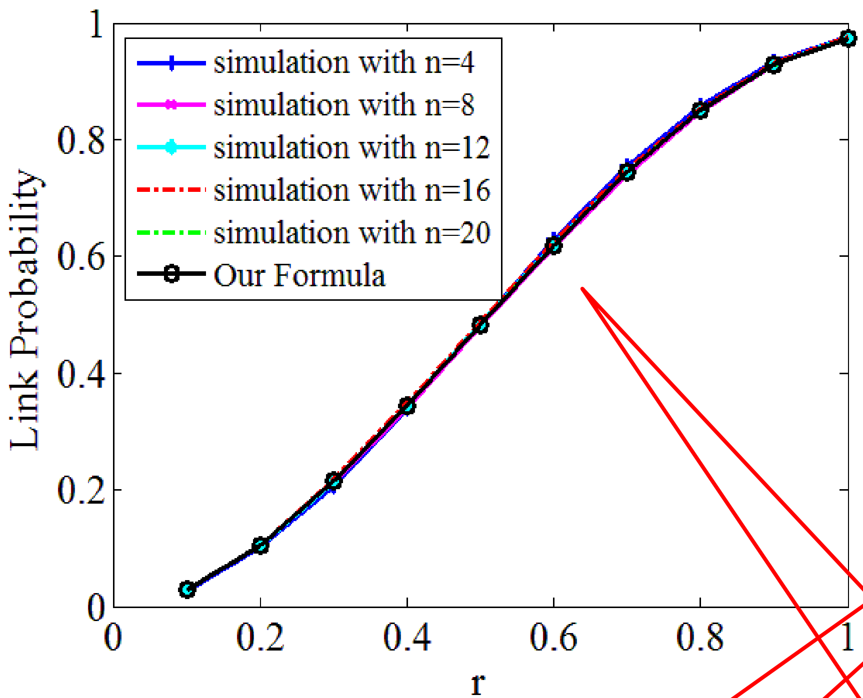
2D



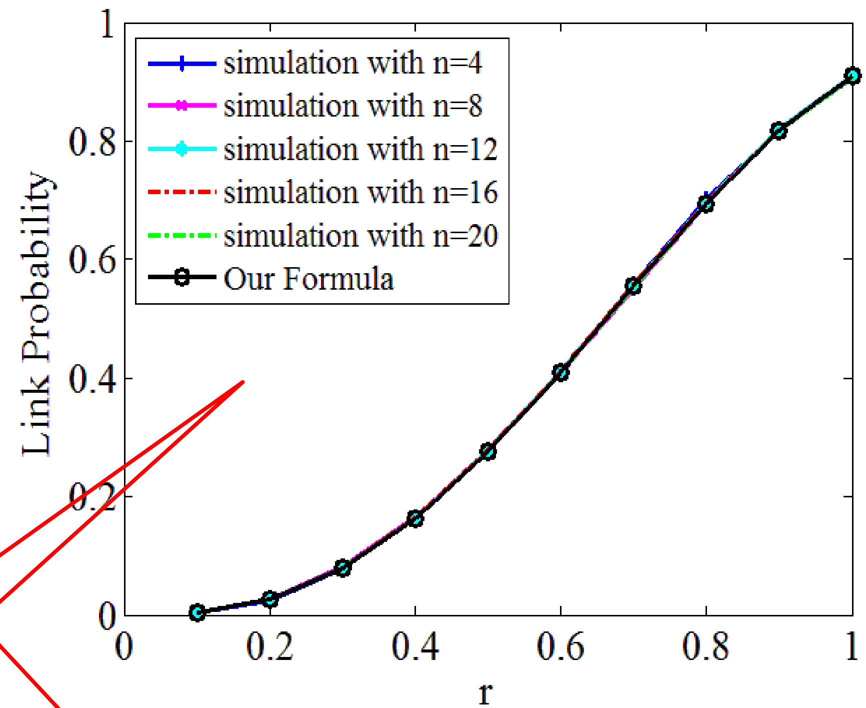
3D

Link probability depends on only r in both 2D and 3D

Test of Link Probability for less nodes



2D



3D

Link probability in 3D is less than 2D

Link probability depends on only r in both 2D and 3D for less number of nodes

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Average Degree

$$E_{ND}(n,r)=(n-1)F_{ND}(r)$$

Expected/ Average degree

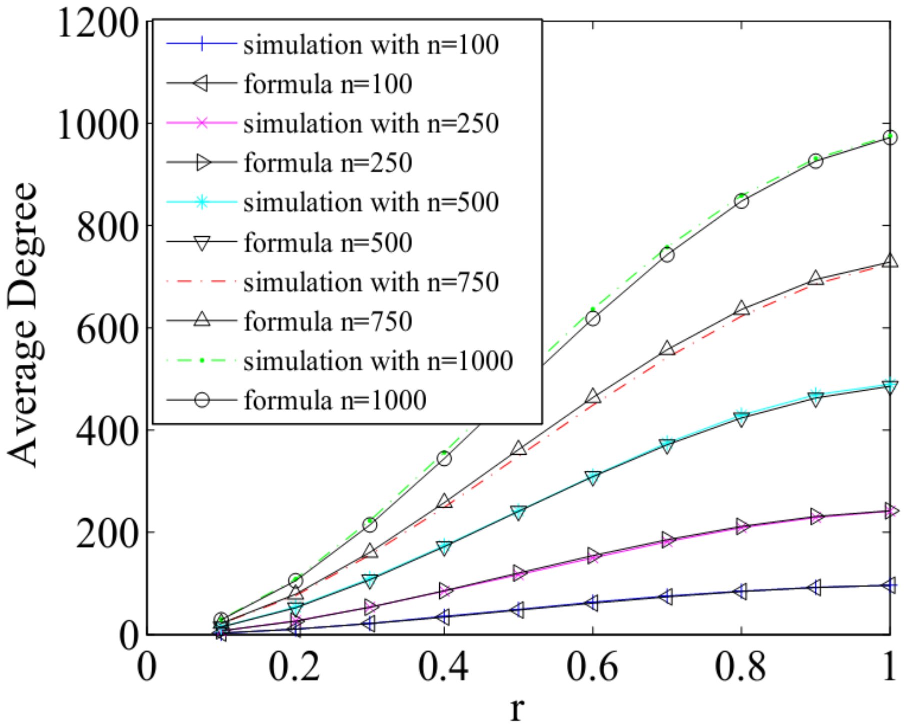
$$E_{2D}(n,r)=(n-1)\left(r^2\left(\frac{r^2}{2}-\frac{8r}{3}+\pi\right)\right)$$

Average Degree for 2D

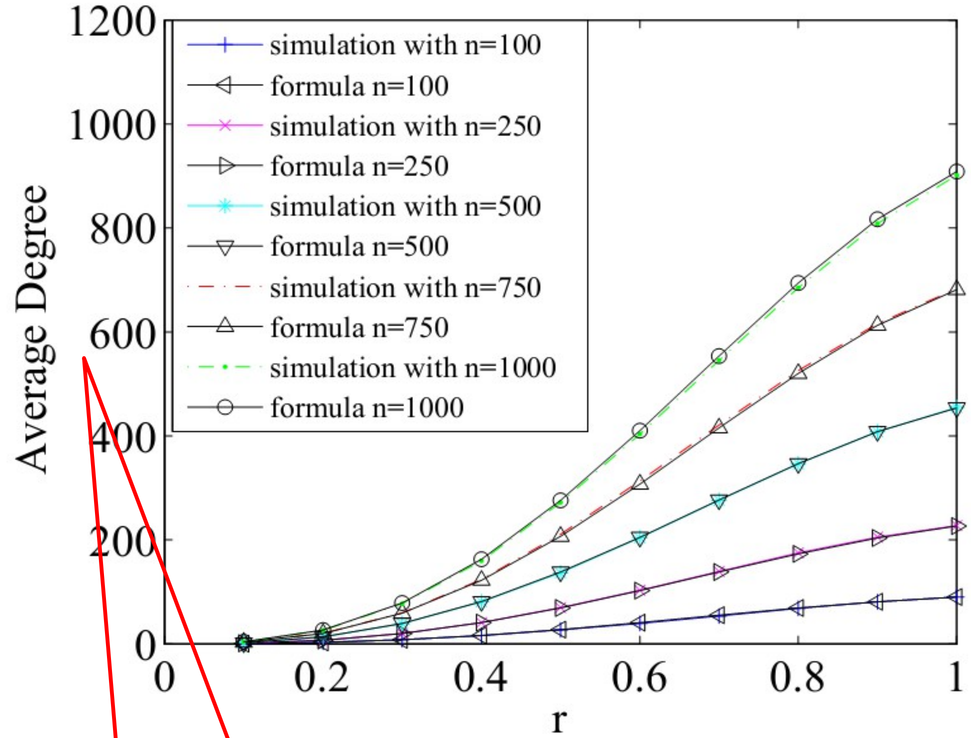
$$E_{3D}(n,r)=(n-1)\left(\frac{4\pi r^3}{3}-\frac{6\pi r^4}{4}+\frac{8r^5}{5}-\frac{r^6}{6}\right)$$

Average Degree for 3D

Test for Average Degree



2D



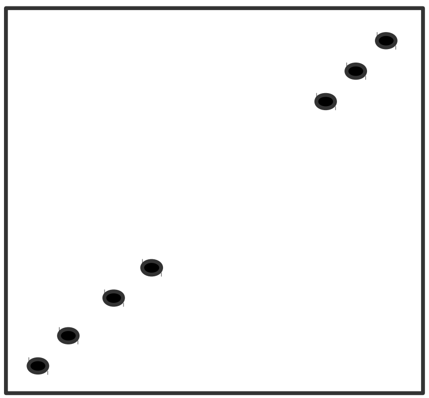
3D

Exactly matches
in 3D and 2D

Diameter

$$Diameter_{ND}(r)=\left\lceil \frac{\sqrt{N}}{r} \right\rceil$$

Diameter when n → ∞



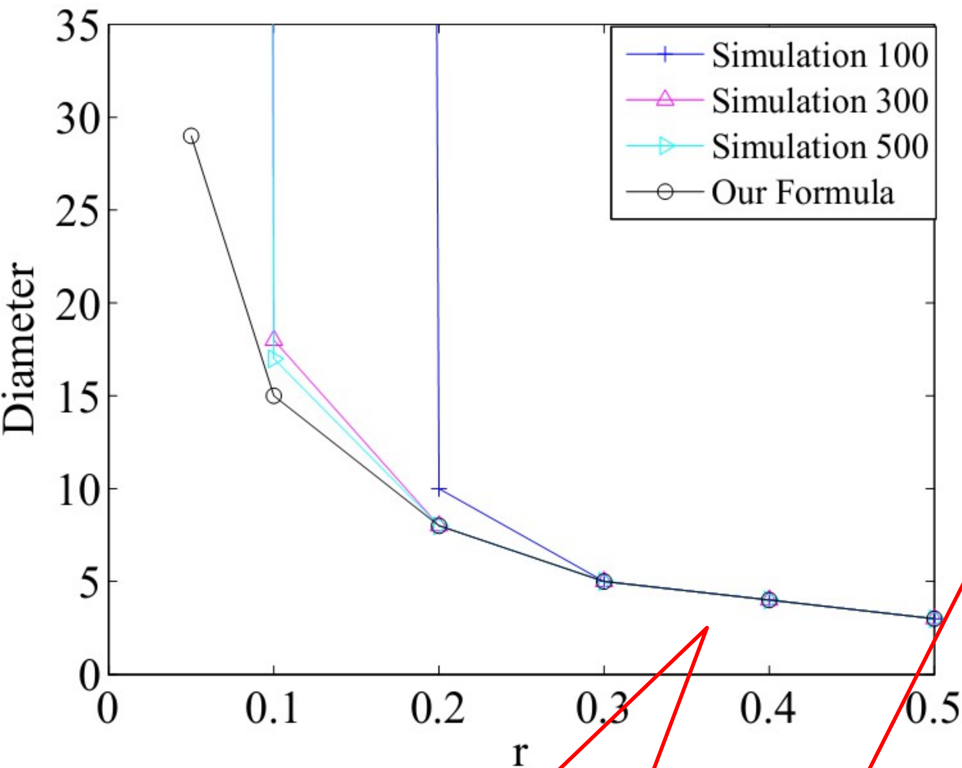
$$Diameter_{\mathrm{2D}}(r)=\left\lceil \frac{\sqrt{2}}{r} \right\rceil$$

Upper bound
Diameter for 2D

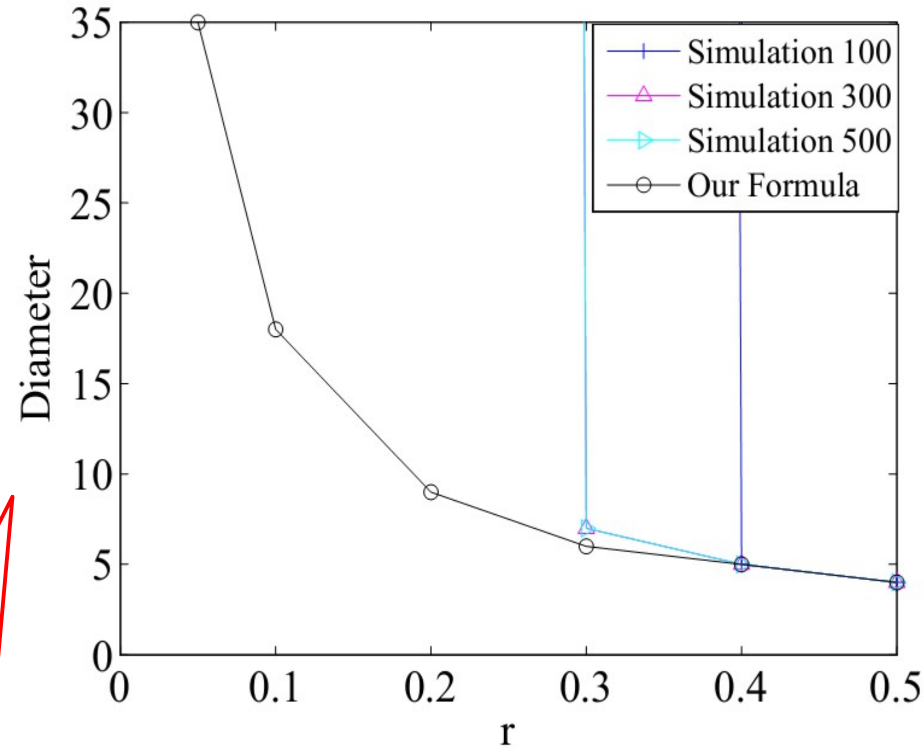
$$Diameter_{\mathrm{3D}}(r)=\left\lceil \frac{\sqrt{3}}{r} \right\rceil$$

Upper bound
Diameter for 3D

Test for Diameter



2D



3D

While r increased approximation getting better

Our approach more effective in 2D and

Average shortest path length

$$Expected\ \ Distance_{ND} = \int_0^{\sqrt{N}} t\, f_{ND}(t)\, dt \quad where\ f_{ND}(t)\ is\ distance\ \ pdf$$

$$E_{hopND} \geq \frac{Expected\ \ Distance_{ND}}{r}$$

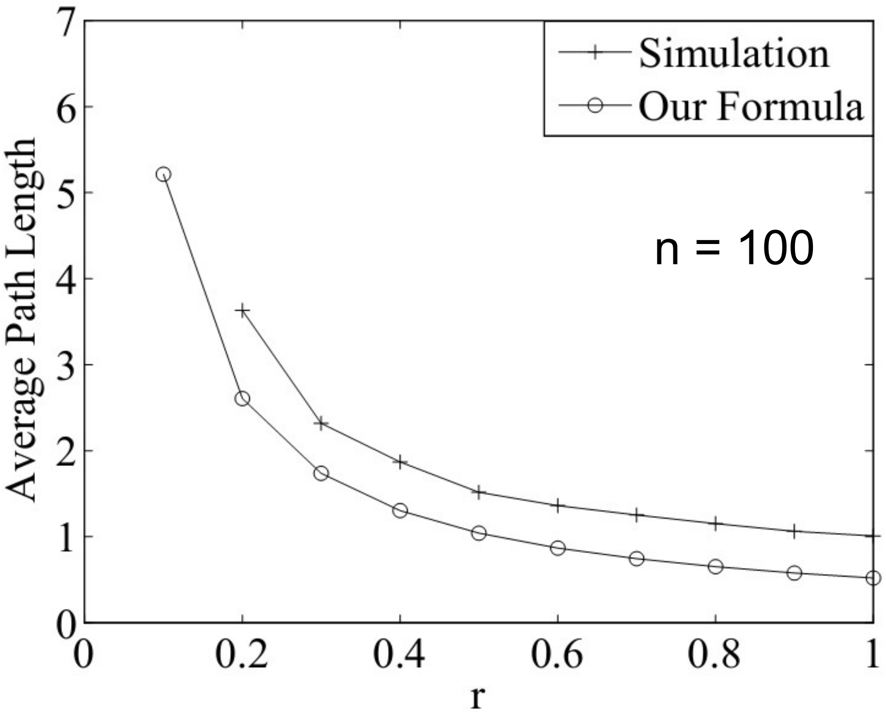
$$E_{hop2D} \geq \frac{0.52140543}{r}$$

0.52.. is expected distance in unit square

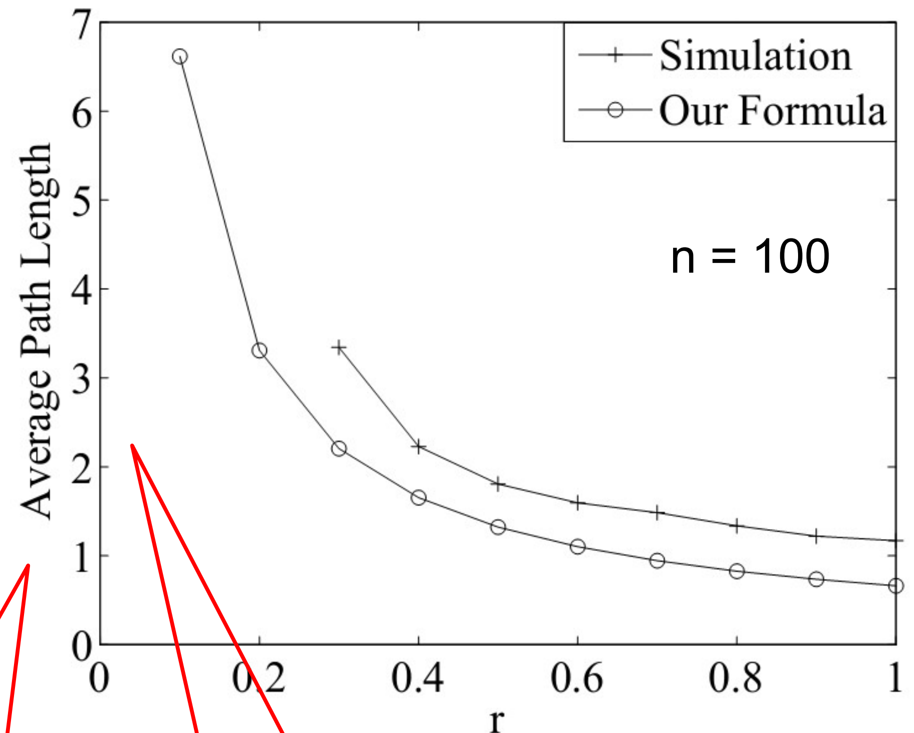
$$E_{hop3D} \geq \frac{0.661707182}{r}$$

0.66.. is expected distance in unit cube

Test for Average shortest path



2D



3D

Approximation and simulation follow similar path

Avg. shortest of 3D higher than 2D

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Conclusion

- **Developed Analytical Formulas for 2D and 3D**
  - Link probability
  - Degree
  - Diameter
  - Average Shortest Path
- **All formulas are verified by Simulation**
- **Studied effects of communication range and number of nodes to Topology in Networks**

Future Work

- **Develop formulas for some other characteristics**
  - Coverage
  - Connectivity
  - Entropy (Randomness of a network)
- **Study Topological Characteristic under motion environment in Wireless Network**

Questions

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