

The Reinvestment Assumption

© Dallas Brozik, Marshall University

Every field of study has its little odd bits, and one of the odd bits in finance is the reinvestment assumption. It is an artifact of the capital budgeting process, it has important implications, it can be avoided quite easily, and most text books explain it wrong. This is how it really works.

The most powerful tool used in finance is the time value of money formula. It is used many places, including some where it really is not appropriate. Perhaps the most important application is in capital budgeting. The basic formula looks like this:

$$NPV_0 = \sum_{t=0}^N \frac{CF_t}{(1+k)^t}$$

The value of the asset right now (time = 0) is the sum of the discounted cash flows that will accrue from the holding of the asset. By having the time line start at t=0, it is not necessary to explicitly consider the price of the asset since the negative cash flow at t=0 is already in the summation. This version of the formula contains all the necessary pieces.

It must be noted that this whole formula only has four parts, the net present value, the future cash flows, the times at which the cash flows are received, and the discount rate k. Since this is a formula that contains an equal sign, you only need to know three of the pieces to be able to calculate the fourth. To make it even easier, the cash flows and the times they occur are calculable. The analyst figures out that the cash flow at time=5 will be \$100, and that is what goes into the equation or on the time line (it's all the same).

Once the cash flows and the times they occur are specified, it is only necessary to specify one of the remaining two variables in order to be able to calculate the fourth. In many cases, the discount rate k is specified. Then there is only a bit of math required to discount all the future cash flows back by the specified rate, add them up, and the total is the net present value. In general, if the net present value (NPV) is greater than 0, the

project more than recovers its costs, so it should be undertaken. Projects with a negative NPV should be avoided since they would result in an overall loss.

NPV calculations are quite easy in concept, though the details may be messy at times. Nevertheless, NPV analysis is quite powerful and provides valuable information that can be used in making capital budgeting decisions. NPV does have one drawback, however, which can lead to problems, and this is the problem of relative scale. Assume that you had done the NPV calculation for a proposed project and found that the NPV is a positive \$1,000,000. At first glance this seems pretty impressive; a million bucks is nothing to be sneezed at. Note that the million dollars is a measure of the absolute value of the benefits of the proposed project since it gives you an absolute dollar value. But there is the embedded question of relative scale; how big was the project? If you invest \$1 today and get back \$1,000,001 tomorrow, you have stumbled on one heck of a good deal (assuming it is legal, but that's a different problem). But if you have to invest \$1,000,000,000,000,000 today to get a really big pile of money sometime in the far distant future and the NPV is only a measly \$1,000,000, the deal might not be so good. Simple NPV calculations net out all the costs and benefits, so it is not possible to see the relative scale of the project just from the final NPV value.

The second approach to the capital budgeting problem comes from break even analysis. If a project has a positive NPV, it is a good deal. If a project has a negative NPV, it is a bad deal. If the project has $NPV=0$, managers should be indifferent to the project since it neither helps nor hurts. By setting the $NPV=0$ in the time value of money formula, it is now possible to solve for the discount rate k that would cause the value of the cash outflows to equal the value of the cash inflows. This discount rate is called the internal rate of return (IRR).

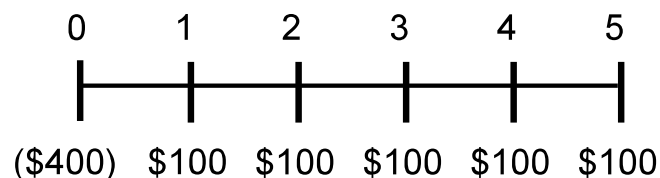
The IRR gives the analysis a relative dimension. For example, you might find that the IRR for a certain project is 15%. If your actual cost of capital is 10%, the investment would yield more than it cost, so it would be a good deal. If your cost of capital is 20%, it would not be wise to invest in a project that only yields a 15% return. Were you to take the

cash flows from the project and discount them at 10%, the NPV would be positive; if you discounted at 20%, the NPV would be negative. In most cases NPV and IRR complement each other and given the same cash flows will result in the same decision.

At this point it would seem logical to ignore NPV and switch to IRR, but IRR has a problem with absolute scale. Assume that your cost of capital is 10%, and you have to choose between two mutually exclusive projects. Project A has an IRR of 100%, and project B has an IRR of 20%. Using strictly IRR analysis, project A would be chosen over project B. But consider the absolute scale of project A: you invest \$1 today and get \$2 tomorrow. With project B you would invest \$1,000,000 today and get \$1,200,000 tomorrow. The NPV of project A is only \$1 while the NPV of project B is \$200,000. IRR fails to recognize the difference in absolute scale, though it does indicate relative scale.

In practice, both NPV and IRR should be used in analyzing potential investments. One gives you a feel for relative size, and the other indicates absolute size. Both are valuable pieces of information in the decision process. In most cases, both NPV and IRR will give the same go/no-go decision. They should; they both derive from the same time value of money formula. It is just a matter of how you look at the problem.

BUT.... There is a little piece of mathematics hiding deep inside the formula that can sometimes rear its ugly head, and this is the reinvestment assumption. Consider the following project. If you invest \$400 today, you will receive \$100 per year at year end for each of the five years. The time line for this project would look like this:



The IRR for this project is 8%. If you were to discount the five cash inflows by 8%, their sum would be a positive \$400 which would just offset the \$400 cost of the project and

result in an NPV of zero. That is what the IRR really is; the discount rate that results in an NPV of zero.

But now the math must be remembered. In order to solve the problem mathematically for the IRR, it is first necessary to expand the summation.

$$0 = \sum_{t=0}^5 \frac{CF_t}{(1+k)^t} = \frac{-400}{(1+k)^0} + \frac{100}{(1+k)^1} + \frac{100}{(1+k)^2} + \frac{100}{(1+k)^3} + \frac{100}{(1+k)^4} + \frac{100}{(1+k)^5}$$

This can be simplified to:

$$0 = -400 + \frac{100}{(1+k)^1} + \frac{100}{(1+k)^2} + \frac{100}{(1+k)^3} + \frac{100}{(1+k)^4} + \frac{100}{(1+k)^5}$$

Now it is time to multiply everything by $(1+k)^5$ to get all that garbage out of the denominators. When you do this you get:

$$0 = -400(1+k)^5 + 100(1+k)^4 + 100(1+k)^3 + 100(1+k)^2 + 100(1+k) + 100$$

At this point it is necessary to expand each of the polynomials, multiply the results by the coefficients, and combine like terms. Do this and you will get a polynomial of the form:

$$0 = A^5 + B^4 + C^3 + D^2 + E + \text{constant}$$

This equation has five possible roots, since the highest powered term is of degree 5. So now you need to look up the general solution to the quintic equation and plug in the coefficients and calculate the five possible roots. There is just one problem, though. There is no general solution to the quintic equation. Never was and never will be (that was proven by a mathematician named Abel about 200 years ago). There are general solutions up to the quartic (highest degree is 4), but those will not help here. This problem cannot be solved with a simple mathematical formula.

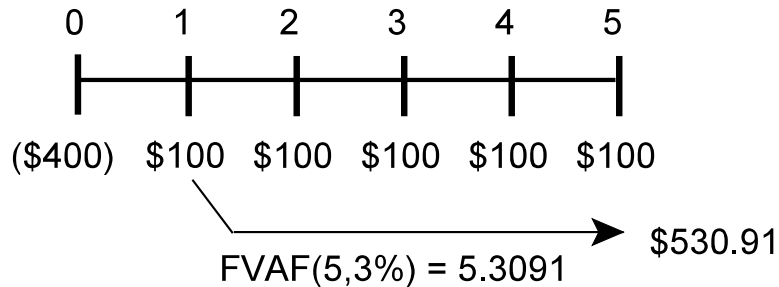
There is a practical way around this problem called interpolation, and that is how such problems are solved. It is exactly like playing the children's game "battleship". You

guess at an answer, and if it is right, great. If it is wrong, guess again. In this case, you make a guess at the appropriate discount rate and solve for the NPV. If you guess 6%, the NPV would be around \$21. Since the NPV is positive, you had not discounted the cash flows by a large enough value, so try again. This time if you guess 10%, you would get an NPV of about -\$21, so you had discounted too much and need to try a lower value. At a discount rate of 8%, you get an NPV of \$0, and this is the IRR.

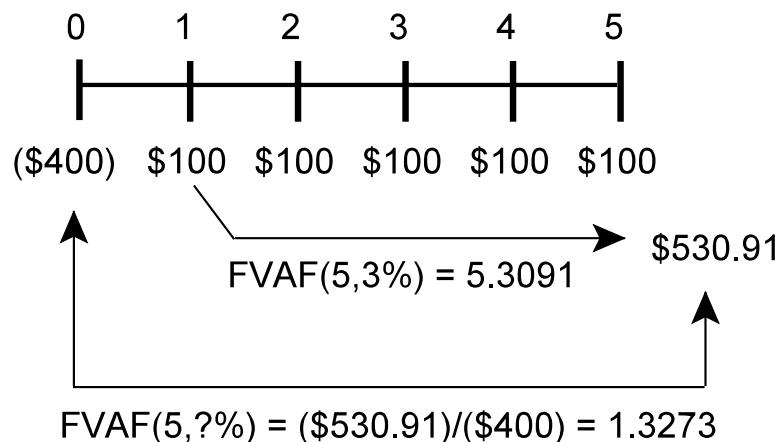
But there is still a difficulty. The original problem was quintic and had five possible roots; 8% is only one of them. There are still four more roots out there somewhere. As it turns out, for this problem all five roots are simultaneous, that is they are all 8%. The reason this is so is because of the pattern of the cash flows. On the time line, the first cash flow at $t=0$ is negative, and then the sign of the cash flow changes and all the other cash flows are positive. Each time the sign changes, a new distinct root enters the solution (that is just the way the math works). Since there was only one change of sign, there is only one distinct root to the problem, in this case 8%. If there are more sign changes, there are more roots, and this really muddies up the water. It is possible to have a problem that has an internal rate of return of -50%, 0%, 25%, and 100%, and these are all mathematically valid solutions. Needless to say, interpretation of such results is difficult. When this happens, just do NPV and get it over with.

And now for the reinvestment assumption. In the simple problem above the IRR is 8% for all five of the possible roots. This means that if you are going to really get an 8% return on this deal, each cash flow must earn 8% for the life of the project. The more common way to state this is that in order to achieve an 8% return on the entire investment, all cash flows must be reinvested at 8% until maturity. No cash flows can be diverted for other purposes. No better investments can be taken should they come along. The cash is essentially tied up for the life of the project, and you **MUST** find projects that will return 8% for the various time horizons each cash flow faces. This could get tricky.

Suppose that you are not able to find other investments that return 8%. Assume that you only have a savings account that pays 3% in which you plan to stick the cash when it comes to you. This is what would happen:



At the end of the project you would have \$530.91. This is a nice number, but it still needs to be converted into a “return”. Note that at the beginning of the project \$400 is invested and by the end of the project this amount has grown to \$530.91. The fact that this money was sitting in your savings account is of no consequence to the analysis, you started with \$400 and ended with \$530.91. It is now only necessary to compare the initial amount with the terminal amount.



For the purposes of the comparison of initial and terminal values, either present or future value calculations can be used. It is easier to use future value factors. In this case, the future value factor that will turn \$400 into \$530.91 is 1.3273. Note that all compounding effects are included in the terminal value. It is not necessary to make any other adjustments to the calculations; everything is already in the terminal value.

In order to use this information to identify a “return”, it is necessary to remember how future value factors are calculated.

$$FVF(N,i\%) = (1 + i)^N$$

with a little mathematics this formula becomes

$$\sqrt[N]{FVF(N,i\%)} = (1 + i)$$

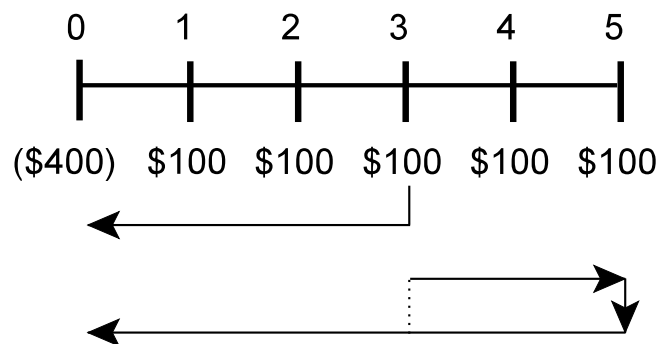
In this case since the future value factor for five periods is 1.3273.

$$\sqrt[5]{1.3273} = (1 + i) = 1.0583$$

There is thus an implied interest rate of 5.83%. This value is called the modified internal rate of return (MIRR). The MIRR captures the actual reinvestment rate and not the implied reinvestment rate of the periodic cash flows.

There is no reinvestment assumption associated with the MIRR because the reinvestment rate is specified. While the simple IRR carries the baggage of the reinvestment assumption, the MIRR does not. The MIRR is therefore probably a better way to measure the implied return from a project and gives a more reasonable measurement for comparison against other projects.

The reinvestment assumption can thus be avoided by using the modified internal rate of return approach to capital budgeting. Textbooks often claim that there is a reinvestment assumption associated with using NPV, but this is simply wrong. It is usually claimed that the NPV approach assumes that all cash flows are reinvested at the discount rate until the project is complete. Consider the following diagram:



In the NPV calculation, the \$100 at time 3 is discounted back to time 0 at the appropriate discount rate. If you were to compound the cash flow two periods to time 5 and then discount it back five periods, the result is the same as a three period discount. You can, if you wish, pretend that there is a reinvestment assumption for NPV calculations, but you are only pretending. There is no requirement in the NPV calculation for reinvestment. In fact, any cash flow can be used as it occurs for any reason, and it still does not affect the NPV calculation. There is no reinvestment assumption for NPV.

The reinvestment assumption is an artifact of the mathematics involved in the IRR calculation. There is nothing that can be done about this, it is simply the mathematics. But there is a real problem here. If there is a really great multi-year project with an IRR of 50%, it is highly unlikely that there will be subsequent projects with a 50% return. Anyone who expects the final audit to show a 50% return will be disappointed. The MIRR technique requires the analyst to specify the reinvestment rate, so there is no assumption. The results of MIRR calculations are probably more likely to occur than the IRR. And remember, NPV has no reinvestment assumption. In capital budgeting calculations, the joint use of NPV and MIRR will probably result in the most complete, accurate analysis, and the whole question of the reinvestment assumption can be avoided.