A General Model for Valuing Equity Securities

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Securities may come in many forms, but all securities contain components that may be described as either debt or equity. While the valuation of debt securities is rather straightforward, the valuation of equity securities can be difficult. The challenge in valuing equity securities is to capture all aspects of that security which represent value to the holder.

The valuation of equity securities must consider all factors relevant to investors and their wealth. While the most common aspect of equity securities is their payment of dividends, another typical characteristic is that the holders of the securities own a portion of the firm, and this ownership can be of value. The book value of the firm, which represents a claim on the firm's assets, may be more important to the shareholder than a token dividend. For example, the stock of a company with a true book value of $1,000 per share that only pays dividends of $1 per share would sell for far more than the few dollars that a strict dividend pricing model would suggest. Such companies could be growth companies with token dividends or "living corpses" that are worth more dead than alive. Valuation models that ignore a firm's book value are necessarily incomplete as are models which do not capture dividends properly.

The traditional dividend discount model proposed by Gordon and Shapiro [3] was a major step in understanding how securities are priced. In practice, however, this model has been less than totally successful. Jacobs and Levy [4] recognize these shortcomings and identify a number of other attributes, including the book/price ratio, that could affect equity pricing. Sorensen and Williamson [5] modify the dividend discount approach by considering various multiperiod models. Estep [1] attempts to find a stock's total return using the company's growth rate, return on equity, and price/book ratio. Fielitz and Muller [2] use growth, yield, and price/earnings ratios to determine a stock's value. Though none of these techniques is totally successful, they all indicate that the traditional dividend discount model is incomplete and that equity valuation could be affected by the firm's book value and the investor's holding period.

Model Development

The starting point for developing a complete equity valuation model is to specify the claim that the shareholder has on the firm. In the broadest definition, a share of stock gives the holder a claim on the assets of the firm and all cash flows that these assets might generate. At any time, the value of the share should be worth at least the current book value per share since this amount represents a claim against existing assets. Stock also gives the shareholder a claim on the future earnings of these assets, and the discounted value of future earnings per share must be added to the value of the stock. Note that the earnings per share and not just the dividends per share are important to the shareholder. Any earnings not paid as dividends are retained by the firm. In this form, these retained earnings augment the book value per share and increase the claim of the shareholder. In mathematical form, the value of a share of equity would be

$$V_0 = BV_0 + PV(\text{Future Cash Inflows})$$

where,

V_0 = Current Value per Share
BV_0 = Current Book Value per Share
PV(Future Cash Flows) = Discounted Value of Future Cash Flows

Anyone purchasing a share of stock must at a minimum pay the current shareholder for the current book value of the stock. If the stock were to sell for less than its true book value, an arbitrage opportunity would exist for speculators to buy all the stock in the company and realize a profit from the liquidation of assets. Stocks usually sell for more than their book value which indicates that some assets like trademarks, market share, and a skilled workforce are not reflected on the balance sheet. These "invisible assets" may not appear on the balance sheet, but the market recognizes their value and prices the stock accordingly. Similarly, there could be "invisible liabilities", like product liability lawsuits, which could depress the market price of a stock below its book value. (In those cases where a stock appears to be selling for less than book value, investors apparently do feel that the "invisible liabilities" exceed the "invisible assets" of the firm.)

Since the shareholder cannot realize the book value implicit in the ownership of a share of stock until the stock is sold, the present value of the initial book value of a share of stock held for N periods would be

$$\text{Present Value of Initial Book Value} = \frac{BV_0}{(1+k)^N}$$

where,

BV_0 = Current Book Value per Share

The book value of the stock is periodically augmented by retained earnings. In order to calculate these additions to book value, it is necessary to determine the amount of retained earnings that will be added each year. Assume that the investor plans to hold the share for N periods. Given a constant growth rate for retained earnings, the per share value of each addition to retained earnings would be

$$RE_t = RE_0 \left(1 + g_{RE}\right)^t$$

where,

RE_t = Retained Earnings per Share for period t
RE_0 = Initial Retained Earnings per Share
g_{RE} = Growth Rate of Retained Earnings

For an investor who holds the stock for N periods, the total addition to the stock's per share book value would be

$$\text{Per Share Addition to Book Value} = \sum_{t=1}^{N} RE_t$$

Since the investor must wait until the stock is sold to realize these gains in book value, only the present value of these increases in book value would be reflected in the stock price. For an investor who plans to hold a share of stock for N periods, the value of the additions to book value per share would be

$$\text{Present Value of Per Share Additions to Retained Earnings} = \frac{\sum_{t=1}^{N} RE_t}{(1+k)^N}$$

A third cash flow received by the investor is cash dividends per share. Since the dividends are received periodically, each dividend must be discounted from the time it is received to determine its component value of the value of the stock.

$$\text{Present Value of Dividends Per Share} = \sum_{t=1}^{N} \frac{D_t}{(1+k)^N}$$

where,

D_t = Per Share Dividend at time t

The final cash flow that the shareholder receives is the price of the stock when it is eventually sold. However, it would be inappropriate to take credit for the entire sales price since it includes the then current book value per share which has already been accounted for by the terms for initial book value and retained earnings. The relevant portion of the selling price is the value of all future cash flows that will accrue to the shareholder who purchases the stock at the future time. These future cash flows would include any future dividends and retained earnings and could also include any market effects that would affect cash flows such as changes in market share or the expiration of patents. The value of this final cash flow could be either positive or negative, depending on the prospects of the firm. Since this final cash flow occurs at a future date, it must be discounted to the present.

$$\text{Present Value of Future Cash Flows} = \frac{FCF_N}{(1+k)^N}$$

where,

FCF_N = Final Cash Flow at time N

The complete model for the valuation of corporate equities contains four terms. The shareholder has a claim on the firm's current book value, any additions to book value that come about through the mechanism of retained earnings, dividends, and the value of certain other future cash flows. The generalized equity valuation model can thus be stated as

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{\sum_{t=1}^{N} RE_t}{(1+k)^N} + \sum_{t=1}^{N} \frac{D_t}{(1+k)^N} + \frac{FCF_N}{(1+k)^N}$$

Given the assumption of constant, continuous growth rates for earnings, retained earnings, and dividends, the value of the stock can be rewritten as shown below (see Appendix I for the derivation). This is a complete equity valuation model that captures all cash flows that accrue to the shareholder and the initial book value of the firm.

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_0}{(1+k)^N}\left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}}\right] + \frac{D_0}{(k - g_D)}\left[(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}\right] + \frac{FCF_N}{(1+k)^N}$$

Note that the investor's holding period does indeed influence the value of the stock as do the growth rates of the individual value components. This implies a clientele effect in the market. Clienteles may form for growth, dividends, or potential future gains, and each group of investors places an appropriate value on a stock. The market price reflects which clientele is willing to pay more for a given share of stock. This explains why some stocks can carry very high P/E ratios. Investors in such stocks may have long investment horizons, expect high growth rates, and/or expect a large contribution to the future cash flow from developments within the firm, such as increases in market share or an improved workforce. Stocks with lower P/E ratios could indicate investors who have shorter investment horizons, expect the firm's position in the market to deteriorate, and/or will accept lower growth rates. There could even be clienteles of investors who prefer growth of retained earnings or growth of dividends.

Applications

In the following examples, it will be assumed that the growth rate of earnings per share, the growth rate of dividends per share, and the growth rate of retained earnings per share are all equal and constant for an equity type security that pays dividends and represents ownership of the firm. (See Appendix II for a detailed discussion of this assumption.) Using the general model, it is possible to value equity securities under various conditions. Though some of the following cases may seem trivial, the model's ability to correctly value these stocks shows the robustness of the model's formulation.

CASE 1:

Normal Growth, Dividend Payment, Infinite Holding Period.

Assume that the stock pays dividends that grow at a constant rate, that the rate of dividend growth is less than the stock's discount rate, and that the stock will be held forever. These are the conditions implicit with the traditional dividend discount model.

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_0}{(1+k)^N}\left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}}\right] + \frac{D_0}{(k-g_D)}\left[(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}\right] + \frac{FCF_N}{(1+k)^N}$$

Due to the infinite holding period, it is necessary to take the limit of each term.

$$\lim_{n\to\infty} \frac{BV_0}{(1+k)^N} = 0$$

$$\lim_{n\to\infty}\frac{RE_0\left[\dfrac{(1+g)^{N+1}-(1+g)}{g}\right]}{(1+k)^N}=\left[\frac{RE_0}{g}\right]\lim_{n\to\infty}\left[\frac{(1+g)^{N+1}-(1+g)}{(1+k)^N}\right]$$

$$=\left[\frac{RE_0}{g}\right]\left[\lim_{n\to\infty}\frac{(1+g)^{N+1}}{(1+k)^N}-\lim_{n\to\infty}\frac{(1+g)}{(1+k)^N}\right]$$

$$=\left[\frac{RE_0}{g}\right]\left[\lim_{n\to\infty}\frac{(1+g)^{N+1}}{(1+k)^N}-0\right]$$

$$=\left[\frac{RE_0}{g}\right]\left[\lim_{n\to\infty}\frac{(1+g)^{N+1}}{(1+k)^N}\right]$$

since k > g

$$=\left[\frac{RE_0}{g}\right]\left[0\right]=0$$

$$\lim_{n\to\infty}D_0\frac{\left[(1+g)-\dfrac{(1+g)^{N+1}}{(1+k)^N}\right]}{(k-g)}=\left[\frac{D_0}{(k-g)}\right]\lim_{n\to\infty}\left[(1+g)-\frac{(1+g)^{N+1}}{(1+k)^N}\right]$$

$$=\left[\frac{D_0}{(k-g)}\right]\lim_{n\to\infty}\left[(1+g)\left(1-\frac{(1+g)^N}{(1+k)^N}\right)\right]$$

$$=\left[\frac{D_0(1+g)}{(k-g)}\right]\lim_{n\to\infty}\left[1-\frac{(1+g)^N}{(1+k)^N}\right]$$

since k > g

$$=\left[\frac{D_0(1+g)}{(k-g)}\right]\left[\lim_{n\to\infty}1-\lim_{n\to\infty}\frac{(1+g)^N}{(1+k)^N}\right]$$

$$=\left[\frac{D_0(1+g)}{(k-g)}\right]\left[1-0\right]$$

$$=\left[\frac{D_0(1+g)}{(k-g)}\right]=\frac{D_1}{(k-g)}$$

$$\lim_{n \to \infty} \frac{FCF_N}{(1+k)^N} = 0$$

In the limit, the book value, retained earnings, and future cash flow components all approach zero since these cash flows will never be realized. The dividend component becomes the standard Gordon-Shapiro dividend discount model. The traditional dividend discount valuation model is thus a special case of the general model. Under these conditions:

$$V_0 = \frac{D_1}{(k - g)}$$

CASE 2:

Continuous Supernormal Growth, Dividend Payment, Infinite Holding Period.

Assume that the stock pays dividends that grow at a constant rate, that the dividend growth rate is greater than the stock's discount rate, and that the stock will be held forever. Under these conditions, the traditional dividend discount model yields a negative value. If a company were to maintain a supernormal growth rate forever, its stock would clearly be worth a very great deal and not the negative amount indicated by the traditional model.

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_0}{(1+k)^N}\left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}}\right] + \frac{D_0}{(k - g_D)}\left[(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}\right] + \frac{FCF_N}{(1+k)^N}$$

It is necessary to take the limits of the various components as the number of periods approaches infinity.

$$\lim_{n \to \infty} \frac{BV_0}{(1+k)^N} = 0$$

$$\lim_{n\to\infty} \frac{RE_0\left[\dfrac{(1+g)^{N+1}-(1+g)}{g}\right]}{(1+k)^N} = \left[\frac{RE_0}{g}\right]\lim_{n\to\infty}\left[\frac{(1+g)^{N+1}-(1+g)}{(1+k)^N}\right]$$

$$= \left[\frac{RE_0}{g}\right]\left[\lim_{n\to\infty}\frac{(1+g)^{N+1}}{(1+k)^N} - \lim_{n\to\infty}\frac{(1+g)}{(1+k)^N}\right]$$

$$= \left[\frac{RE_0}{g}\right]\left[\lim_{n\to\infty}\frac{(1+g)^{N+1}}{(1+k)^N} - 0\right]$$

$$= \left[\frac{RE_0}{g}\right]\left[\lim_{n\to\infty}\frac{(1+g)^{N+1}}{(1+k)^N}\right]$$

since k < g

$$= \left[\frac{RE_0}{g}\right][\infty] = \infty$$

$$\lim_{n\to\infty} D_0 \frac{\left[(1+g)-\dfrac{(1+g)^{N+1}}{(1+k)^N}\right]}{(k-g)} = \left[\frac{D_0}{(k-g)}\right]\lim_{n\to\infty}\left[(1+g)-\frac{(1+g)^{N+1}}{(1+k)^N}\right]$$

$$= \left[\frac{D_0}{(k-g)}\right]\lim_{n\to\infty}\left[(1+g)\left(1-\frac{(1+g)^N}{(1+k)^N}\right)\right]$$

$$= \left[\frac{D_0(1+g)}{(k-g)}\right]\lim_{n\to\infty}\left[1-\frac{(1+g)^N}{(1+k)^N}\right]$$

$$= \left[\frac{D_0(1+g)}{(k-g)}\right]\left[\lim_{n\to\infty} 1 - \lim_{n\to\infty}\frac{(1+g)^N}{(1+k)^N}\right]$$

since k < g

$$= \left[\frac{D_0(1+g)}{(k-g)}\right][1-\infty]$$

since k < g

$$= \left[\frac{D_0(1+g)}{(k-g)}\right](-\infty) = +\infty$$

$$\lim_{n \to \infty} \frac{FCF_N}{(1+k)^N} = 0$$

The book value is never recovered and the stock is never sold, so both these components are zero. But both the retained earnings and dividend components become infinite. As expected, this type of stock would be quite valuable.

Logically, such a company could not exist. Eventually, a company growing at a supernormal rate would become the entire economy (much in the way a colony of mold grows to the limits of a petri dish). When such a company becomes the economy, the term supernormal becomes meaningless. If some firm were able to grow until it subsumed every other firm and became the economy, there is still the question about the cost of capital relative to the growth rate of the firm. Since the providers of equity funds demand a real return on their investment, the cost of capital would be increased beyond the underlying growth rate of the economy and thus the growth rate of the firm. At this point the conditions for the use of the basic valuation model would obtain.

For cases where the supernormal growth period is finite, the valuation of the firm can be found by applying the basic valuation equation twice. The first period would cover the finite supernormal period. The form of the equation for the second period would depend upon the assumptions concerning the total proposed holding period.

The following cases assume that shareholders have a finite investment horizon of N periods. This assumption is not a restriction but rather a more accurate modeling of investor behavior. Most investors do not plan to live forever or to leave all their holdings to their heirs. These cases will illustrate how the general model can be applied in situations where traditional dividend discount models fail.

CASE 3:

Normal Growth, Dividend Payment, Discount Rate Equals Growth Rate, Finite Holding Period.

Assume that the stock pays dividends that grow at a constant rate, that the dividend growth rate is equal to the stock's discount rate, and the stock will be held for N periods. Under the traditional dividend discount model (CASE 1), the denominator of the fraction becomes 0, and the value of the stock becomes infinite. This result of the traditional model is clearly nonsensical.

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_0}{(1+k)^N}\left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}}\right]$$

$$+ \frac{D_0}{(k - g_D)}\left[(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}\right] + \frac{FCF_N}{(1+k)^N}$$

It is necessary to take the limits of the various components as the growth rate approaches the discount rate.

$$\lim_{g \to k} \frac{BV_0}{(1+k)^N} = \frac{BV_0}{(1+k)^N}$$

$$\lim_{g \to k} \frac{\sum_{t=1}^{N} RE_t}{(1+k)^N} = \left[\frac{1}{(1+k)^N}\right] \lim_{g \to k}\left[\sum_{t=1}^{N} RE_0 (1+g)^t\right]$$

$$= \left[\frac{RE_0}{(1+k)^N}\right] \lim_{g \to k}\left[\sum_{t=1}^{N} (1+g)^t\right]$$

$$= \frac{RE_0 (1+k)\left[\frac{(1+k)^N - 1}{k}\right]}{(1+k)^N}$$

$$= \frac{RE_1\left[\frac{(1+k)^N - 1}{k}\right]}{(1+k)^N}$$

$$\lim_{g \to k}\left[\sum_{t=1}^{N}\frac{D_t}{(1+k)^t}\right] = \lim_{g \to k}\left[\sum_{t=1}^{N}\frac{D_0(1+g)^t}{(1+k)^t}\right] = D_0 \lim_{g \to k}\left[\sum_{t=1}^{N}\frac{(1+g)^t}{(1+k)^t}\right] = D_0 N$$

$$\lim_{g \to k}\frac{FCF_N}{(1+k)^N} = \frac{FCF_N}{(1+k)^N}$$

The initial book value term is merely discounted for the number of periods held. The retained earnings component shows a familiar formula. Since the retained earnings are being "stored" for the future, they represent an annuity. Though retained earnings do not draw interest, they do grow each period. The growth rate acts as a substitute for compound interest. Thus the value of the retained earnings component is merely the future value of an annuity whose first payment is RE_1 received after N periods, discounted back to the present. The dividend term reflects that the present value of a cash stream that is growing by its discount rate is a constant because the growth and discount rates are equal. Since each dividend has the same present value, the total value of the stream is simply the number of dividends received. The value of the future cash flows is not received until after the stock is sold at time N, so this term is discounted for N periods. Under these conditions:

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_1\left[\frac{(1+k)^N - 1}{k}\right]}{(1+k)^N} + D_0 N + \frac{FCF_N}{(1+k)^N}$$

CASE 4:

Zero Growth, No Dividend Payment, Finite Holding Period.

Assume that the stock does not pay dividends, has a zero growth rate, and is held for N periods.

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_0}{(1+k)^N}\left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}}\right]$$
$$+ \frac{D_0}{(k - g_D)}\left[(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}\right] + \frac{FCF_N}{(1+k)^N}$$

The dividend term becomes zero since no dividend is paid, and the solution becomes:

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_0}{(1+k)^N}\left[\frac{(1+g)^N - (1+g)}{g}\right] + \frac{FCF_N}{(1+k)^N}$$

Before taking the limit, it is necessary to simplify the polynomial in the retained earnings term.

$$(1+g)^{N+1} = g^{N+1} + (N+1)\,g^N + \frac{(N+1)(N)}{2}\,g^{N-1} + \frac{(N+1)(N)(N-1)}{6}\,g^{N-2} + \dots + \frac{(N+1)(N)(N-1)}{6}\,g^3 + \frac{(N+1)(N)}{2}\,g^2 + (N+1)\,g + 1$$

$$(1+g)^{N+1} - (1+g) = g^{N+1} + (N+1)\,g^N + \frac{(N+1)(N)}{2}\,g^{N-1} + \frac{(N+1)(N)(N-1)}{6}\,g^{N-2} + \dots + \frac{(N+1)(N)(N-1)}{6}\,g^3 + \frac{(N+1)(N)}{2}\,g^2 + Ng$$

$$\frac{(1+g)^{N+1} - (1+g)}{g} = g^N + (N+1)\,g^{N-1} + \frac{(N+1)(N)}{2}\,g^{N-2} + \frac{(N+1)(N)(N-1)}{6}\,g^{N-3} + \dots + \frac{(N+1)(N)(N-1)}{6}\,g^2 + \frac{(N+1)(N)}{2}\,g + N$$

When g=0, all the terms except the final constant become zero, so:

$$\lim_{g \to 0}\left[\frac{(1+g)^{N+1} - (1+g)}{g}\right] = N$$

Even a stock in such a stagnant company has some value. The book value component is again discounted for the number of periods held. Since the retained earnings do not grow, this component is the discounted value of N additions of a constant amount. There is no dividend component, and the final selling price is discounted for N periods. Under these conditions:

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{(RE_0)(N)}{(1+k)^N} + \frac{FCF_N}{(1+k)^N}$$

If it were expected that the firm would cease operations after N periods, the FCF_N term representing the value of the future cash flows would be zero. But even in a case like this, shareholders would have a claim represented by the initial book value as augmented by additions to retained earnings.

CASE 5:

Zero Growth, No Dividend Payment.

Assume the stock has no earnings and thus pays no dividends or retains any earnings. No Appendix is necessary for this evaluation since the results can be observed directly from the general model. The retained earnings and dividend components both become zero, and the only value in the stock is the book value and the discounted future cash flows. The longer the investor plans to hold the stock, the less the initial book value term is worth. Where such stocks could be interesting is if they represent takeover candidates. If it is assumed that a buyer will pay some high future price, for whatever reason, investors might be willing to make a market for such a stock. The prices of such stocks reflect the market's opinion of the value of the firm to an outside buyer, how long until the stock is purchased, and how much that buyer would pay at that time. Under these conditions:

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{FCF_N}{(1+k)^N}$$

CASE 6:

Normal Growth, No Dividend Payment, Finite Holding Period.

Assume that the stock pays no dividends, has a positive growth rate, and will be held for N periods. This is a simple application of the general model. The dividend term becomes zero. Investors pay for the discounted book value, for the potential accrual of retained earnings, and for the discounted future cash flows of the stock. Under these conditions:

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{RE_0}{(1+k)^N}\left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}}\right] + \frac{FCF_N}{(1+k)^N}$$

This value reflects the average holding period and the consensus growth rate for the firm.

CASE 7:

Normal Growth, Retention of Earnings for a Finite Holding Period, Surrender of the Business.

This rather restrictive type of security could be developed in countries that feared the takeover of local companies and resources by foreign investors. Such a country might invite foreign investors to local develop natural resources but require that the earnings be reinvested in the local economy until a time N when all retained earnings could be withdrawn and the business surrendered to local interests. Even under these conditions, there is some value to the investment. This is actually a modification of CASE 6, and under these conditions:

$$V_0 = \frac{RE_0}{(1+k)^N}\left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}}\right]$$

CASE 8:

Normal Growth, Total Payout of Earnings, Finite Holding Period.

Assume the firm pays out all earnings as dividends, has a positive growth rate, and that the stock will be held for N periods. Since no earnings are retained, the retained earnings term in the general model drops out. The value of the stock is the discounted value of the initial book value component, the discounted value of each of the dividends, and the discounted future cash flows. Note that if the holding period is allowed to become infinite, the book value and final price terms go to zero in the limit, and the result is the dividend discount model, as was shown in CASE 1. Under these conditions

$$V_0 = \frac{BV_0}{(1+k)^N} + \frac{D_0}{(k - g_D)}\left[(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}\right] + \frac{FCF_N}{(1+k)^N}$$

Summary

The general equity valuation model presented here is consistent with existing equity valuation models and is able to analyze other equity valuation situations. The major theoretical improvements of this model are that it includes book value and allows for the explicit inclusion or exclusion of specific cash flows. Due to its general nature, the model can be used to value equity type securities that currently or may in the future exist in different countries.

Applications of the general model include cases of supernormal growth rates, retention of all earnings, and finite holding periods. In these aspects, the model provides a more accurate valuation of such "abnormal" stocks. The general model also provides results in situations where the traditional dividend discount models break down, such as when the discount rate is equal to the growth rate. The inclusion of a finite holding period can be used to value situations in which assets must ultimately be surrendered to a local authority.

References

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APPENDIX I

$$\frac{\sum_{t=1}^{N} RE_t}{(1+k)^N} = \frac{\sum_{t=1}^{N} RE_0 (1+g_{RE})^t}{(1+k)^N} = \frac{RE_0}{(1+k)^N} \sum_{t=1}^{N} (1+g_{RE})^t$$

$$= \frac{RE_0}{(1+k)^N} \left[\frac{(1+g_{RE})^{N+1} - (1+g_{RE})}{g_{RE}} \right]$$

If k = g:

$$\sum_{t=1}^{N} \frac{D_t}{(1+k)^t} = \sum_{t=1}^{N} \frac{D_0 (1+g_D)^t}{(1+k)^t} = D_0 \sum_{t=1}^{N} \left(\frac{1+g_D}{1+k} \right)^t$$

If k … g

$$\sum_{t=1}^{N} \left(\frac{1+g_D}{1+k} \right)^t = N \quad \text{and} \quad \sum_{t=1}^{N} \frac{D_t}{(1+k)^t} = D_0 N$$

$$\sum_{t=1}^{N} \left(\frac{1+g_D}{1+k} \right)^t = \frac{(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}}{(k - g_D)}$$

and

$$\sum_{t=1}^{N} \frac{D_t}{(1+k)^t} = D_0 \left[\frac{(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N}}{(k - g_D)} \right]$$

$$= \frac{D_0}{(k - g_D)} \left[(1+g_D) - \frac{(1+g_D)^{N+1}}{(1+k)^N} \right]$$

APPENDIX II

Under the assumptions of continuous compounding, the following are definitions:

$$E_t = E_0\ e^{g_E t} \qquad D_t = D_0\ e^{g_D t} \qquad RE_t = RE_0\ e^{g_{RE} t}$$

$$E_0 = D_0 + RE_0 \qquad E_t = D_t + RE_t$$

CASE 1: $g_E = g_D = g$

$$E_0 e^{gt} = D_0 e^{gt} + RE_0 e^{gt}$$

$$E_0 e^{gt} - D_0 e^{gt} = RE_0 e^{gt}$$

$$\left(E_0 - D_0\right) e^{gt} = RE_0 e^{g_{RE} t}$$

but

$$E_0 - D_0 = RE_0$$

$$e^{gt} = e^{g_{RE} t}$$

$$\Rightarrow\ g_{RE} = g_E = g_D = g$$

CASE 2: Constant Payout Ratio, D/E = b

$$E_0 e^{g_E t} = D_0 e^{g_D t} + RE_0 e^{g_{RE} t}$$

but

$$D_t = bE_t = bE_0 e^{g_E t}$$

thus

$$E_0 e^{g_E t} = bE_0 e^{g_E t} + RE_0 e^{g_{RE} t}$$

$$\left(1 - b\right) E_0 e^{g_E t} = RE_0 e^{g_{RE} t}$$

but

$$\left(1 - b\right) E_0 = RE_0$$

thus

$$e^{g_E t} = e^{g_{RE} t}$$

$$\Rightarrow\ g_{RE} = g_E = g_D = g$$

CASE 3: $g_E > g_D$

$$E_0 e^{g_E t} = D_0 e^{g_D t} + RE_0 e^{g_{RE} t}$$

$$E_0 e^{g_E t} - D_0 e^{g_D t} = RE_0 e^{g_{RE} t}$$

$$\frac{E_0 e^{g_E t} - D_0 e^{g_D t}}{RE_0} = e^{g_{RE} t}$$

$$\ln\left[\frac{E_0 e^{g_E t} - D_0 e^{g_D t}}{RE_0}\right] = g_{RE} t$$

$$g_{RE} = \frac{\ln\left[\frac{E_0 e^{g_E t} - D_0 e^{g_D t}}{RE_0}\right]}{t}$$

If periodic compounding were used, the growth rate would be

$$g_{RE} = \sqrt[N]{\frac{E_0 \left(1+g_{RE}\right)^N - D_0 \left(1+g_D\right)^N}{RE_0}} - 1$$

Note that under these conditions, the growth rate of retained earnings varies with time. This means that the boundary condition of stable growth rates that was assumed in the general solution has been violated and that the general solution cannot be used under these conditions.

CASE 4: $g_E < g_D$

Recall from CASE 3 that

$$g_{RE} = \frac{\ln\left[\frac{E_0 e^{g_E t} - D_0 e^{g_D t}}{RE_0}\right]}{t}$$

This case is limited at the point

$$E_0 e^{g_E t} = D_0 e^{g_D t}$$

since the dividends cannot exceed earnings. At this limit

$$\ln E_0 + g_E t = \ln D_0 + g_D t$$

The longest time that such an unstable condition can exist is

$$t = \frac{\ln E_0 - \ln D_0}{g_D - g_E}$$

When periodic compounding is used instead of continuous compounding, the results of CASES 1 and 2 remain the same. As in CASE 3, however, the result becomes

$$g_{RE} = \sqrt[N]{\frac{E_0 \left(1 + g_{RE}\right)^N - D_0 \left(1 + g_D\right)^N}{RE_0}} - 1$$

The maximum number of periods that the growth rate of dividends can exceed the growth rate of earnings, as shown in CASE 4, becomes

$$N = \frac{\ln E_0 - \ln D_0}{\ln\left(1 + g_D\right) - \ln\left(1 + g_E\right)}$$

As in CASE 3, CASE 4 violates the boundary conditions used in determining the general solution, and the general solution would not be valid under such conditions.
